

Black-Scholes Model

Option pricing fundamentals

Christos Galerakis

2026-01-13

Table of contents

| | |
|------------------------------------|----------|
| 1 Abstract | 1 |
| 2 Assumptions | 1 |
| 3 The Black-Scholes Formula | 2 |
| 4 Compute (Python) | 2 |
| 5 Put-Call Parity | 2 |
| 6 Option Price Surface | 2 |
| 7 Price vs Stock Price | 4 |
| 8 Sensitivity to Parameters | 5 |
| 9 Time Decay | 6 |
| 10 Option Pricing Table | 6 |
| 11 Limitations | 6 |
| 12 Conclusion | 6 |

1 Abstract

The Black-Scholes model, published by Fischer Black and Myron Scholes in 1973, provides a closed-form solution for pricing European-style options (Black & Scholes, 1973). The model assumes geometric Brownian motion for the underlying asset and derives option prices through a no-arbitrage argument. Despite its simplifying assumptions, Black-Scholes remains the foundation of options pricing and risk management.

2 Assumptions

The Black-Scholes model assumes:

1. Stock price follows geometric Brownian motion with constant μ and σ
2. Risk-free rate r is constant
3. No dividends during the option's life
4. European-style exercise (only at expiration)
5. No transaction costs or taxes
6. Continuous trading is possible
7. No arbitrage opportunities exist

3 The Black-Scholes Formula

Call option price:

$$C = S_0 N(d_1) - K e^{-rT} N(d_2)$$

Put option price:

$$P = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where:

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- S_0 = current stock price
- K = strike price
- T = time to expiration (years)
- r = risk-free rate
- σ = volatility
- $N(\cdot)$ = cumulative normal distribution function

4 Compute (Python)

```
Stock Price (S): $100
Strike Price (K): $100
Time to Expiration: 3 months
Risk-Free Rate: 5.0%
Volatility: 20.0%
```

```
Call Price: $4.61
Put Price: $3.37
```

5 Put-Call Parity

A fundamental relationship links call and put prices:

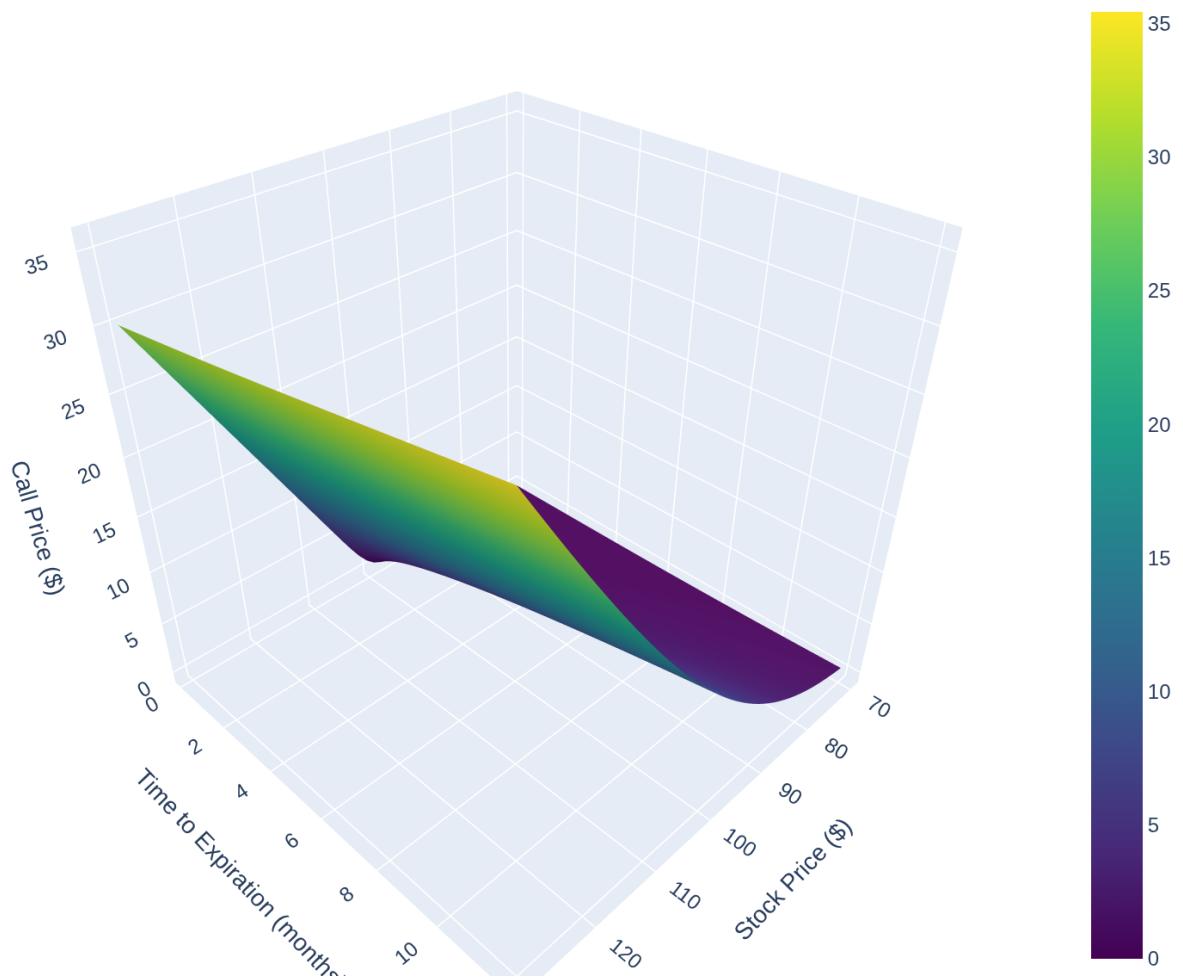
$$C - P = S_0 - K e^{-rT}$$

```
C - P = $1.2422
S - Ke^(-rT) = $1.2422
Difference: $0.000000
```

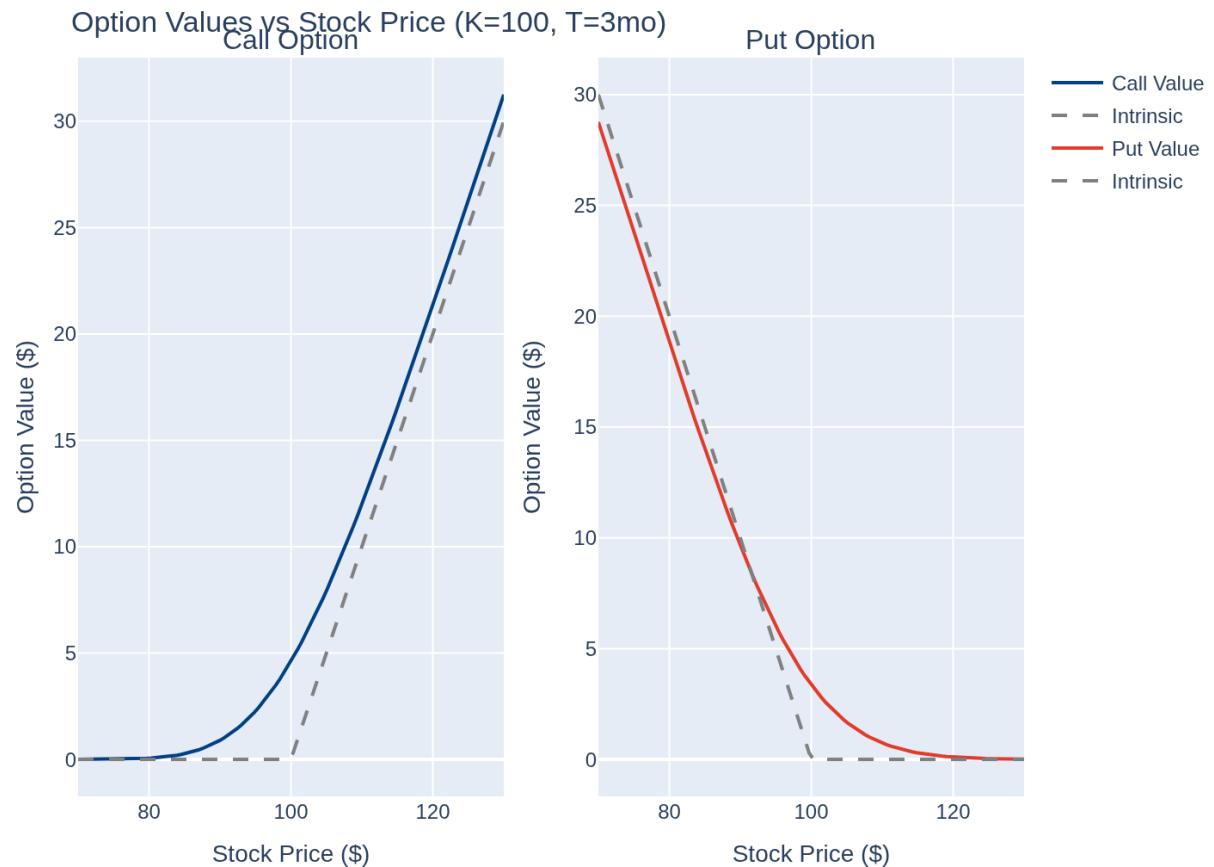
6 Option Price Surface

Option prices vary with stock price and time to expiration.

Call Option Price Surface ($K=100$, $\sigma=20\%$)

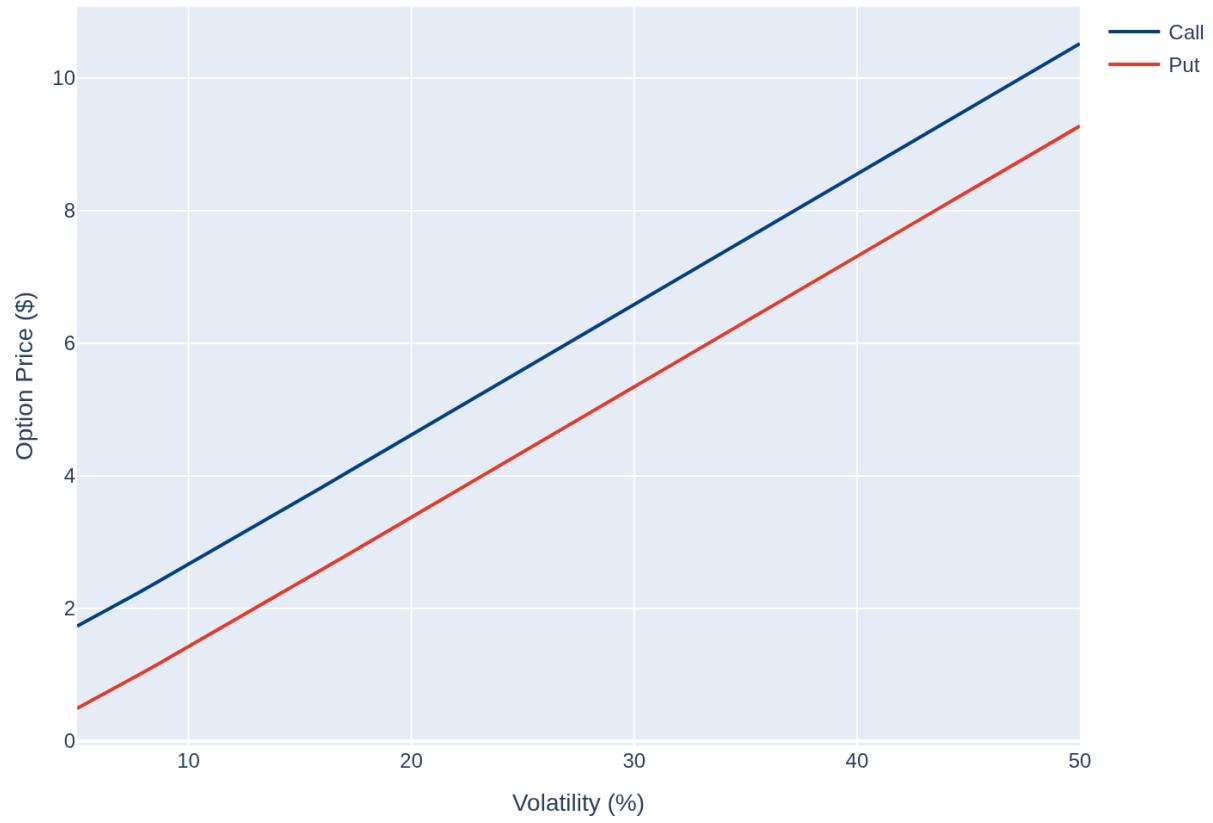


7 Price vs Stock Price



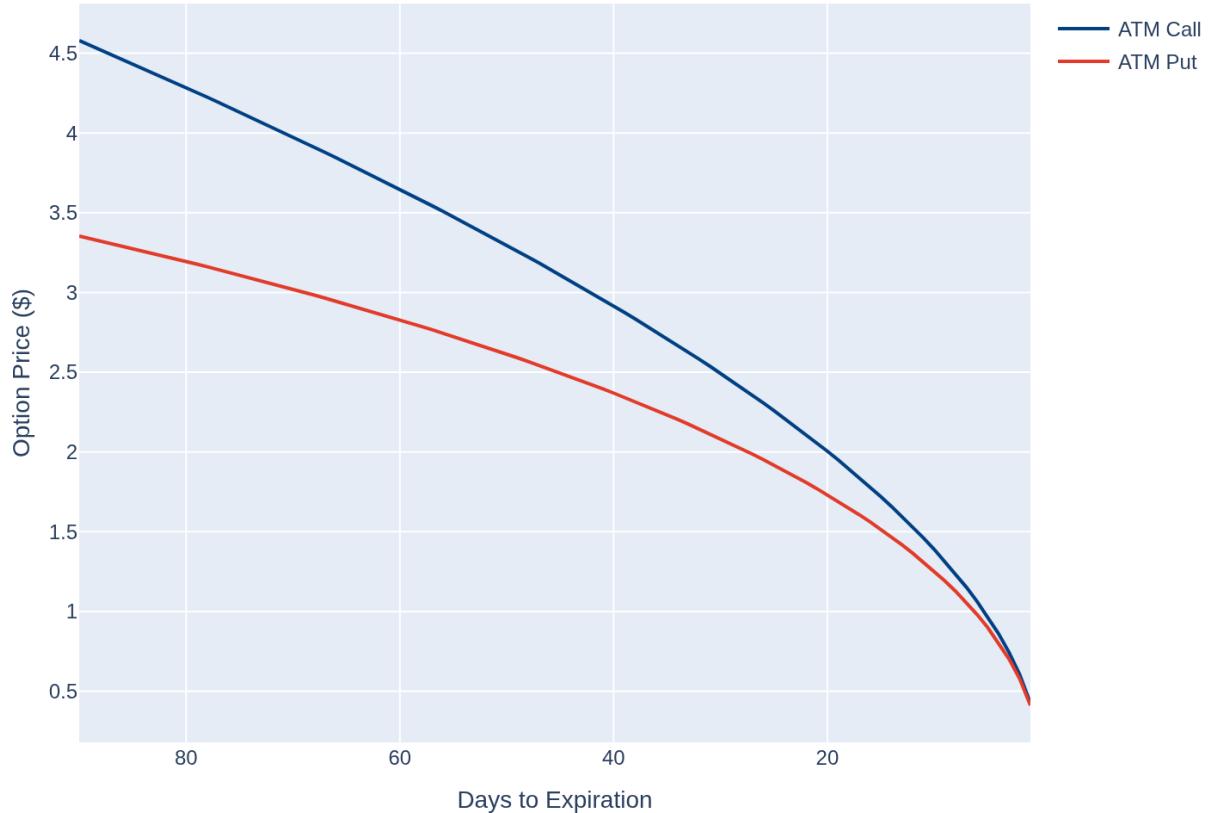
8 Sensitivity to Parameters

Option Prices vs Volatility (S=100, K=100)



9 Time Decay

Time Decay (ATM options, $S=K=100$)



10 Option Pricing Table

| Days Strike | 30 | 60 | 90 | 180 |
|----------------|-------|-------|-------|-------|
| 90 | 10.43 | 11.02 | 11.64 | 13.45 |
| 95 | 5.88 | 6.84 | 7.68 | 9.82 |
| 100 | 2.49 | 3.65 | 4.58 | 6.83 |
| 105 | 0.73 | 1.64 | 2.45 | 4.53 |
| 110 | 0.14 | 0.61 | 1.17 | 2.86 |

11 Limitations

- **Constant volatility:** Real volatility changes over time and varies by strike (volatility smile)
- **No dividends:** Requires adjustment for dividend-paying stocks
- **European only:** Cannot price American options with early exercise
- **Log-normal prices:** Underestimates probability of extreme moves
- **Continuous trading:** Markets have discrete trading and gaps

Despite these limitations, Black-Scholes provides essential intuition for options pricing and serves as a benchmark for more sophisticated models.

12 Conclusion

The Black-Scholes model revolutionized derivatives pricing by providing a closed-form solution for European options. Understanding its components—the relationship between underlying price, strike,

time, volatility, and interest rates—is fundamental to options trading and risk management. While modern practitioners use more sophisticated models, Black-Scholes remains the starting point for options education and a benchmark for model comparison.

References

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637–654.