Stationarity

Time series stability testing

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1 Abstract

A stationary time series has statistical properties (mean, variance, autocorrelation) that don't change over time. Most financial prices are non-stationary, but returns are typically stationary. Testing for stationarity using the Augmented Dickey-Fuller (ADF) test is essential before applying many time series models.

2 Definition

A time series $\{X_t\}$ is **strictly stationary** if the joint distribution of $(X_{t_1}, ..., X_{t_k})$ is identical to $(X_{t_1+h}, ..., X_{t_k+h})$ for all h.

Weak stationarity (more practical) requires:

1. Constant mean: $E[X_t] = \mu$ for all t
2. Constant variance: $Var(X_t) = \sigma^2$ for all t
3. Autocovariance depends only on lag: $Cov(X_t, X_{t+h}) = \gamma(h)$

3 Why Stationarity Matters

- **Predictability**: Non-stationary series have unpredictable statistical properties
- **Model validity**: ARMA, GARCH models assume stationarity
- **Spurious regression**: Regressing non-stationary series produces misleading results
- **Mean reversion**: Stationary series revert to their mean; non-stationary series don't

4 Unit Root and Random Walk

A **random walk** is a classic non-stationary process:

$$X_t = X_{t-1} + \varepsilon_t$$

This has a **unit root** (coefficient = 1). The variance grows without bound over time:

$$Var(X_t) = t \cdot \sigma_\varepsilon^2$$

5 Augmented Dickey-Fuller Test

The ADF test checks for a unit root:

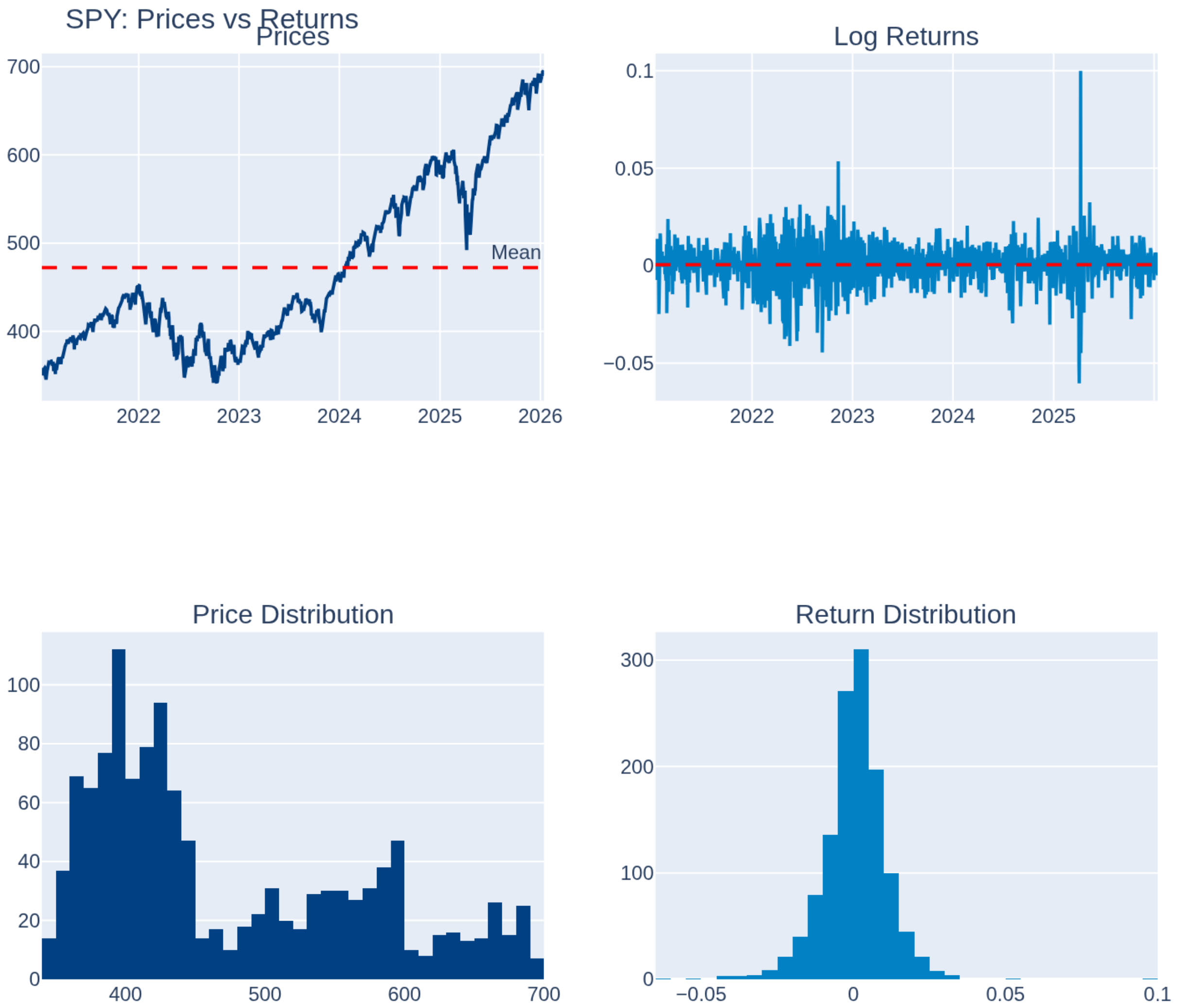
$$\Delta X_t = \alpha + \beta t + \gamma X_{t-1} + \sum_{i=1}^{p} \delta_i \Delta X_{t-i} + \varepsilon_t$$

- **Null hypothesis**: $\gamma = 0$ (unit root exists, non-stationary)
- **Alternative**: $\gamma < 0$ (stationary)
- **Decision**: Reject null if test statistic < critical value

6 Compute (Python)

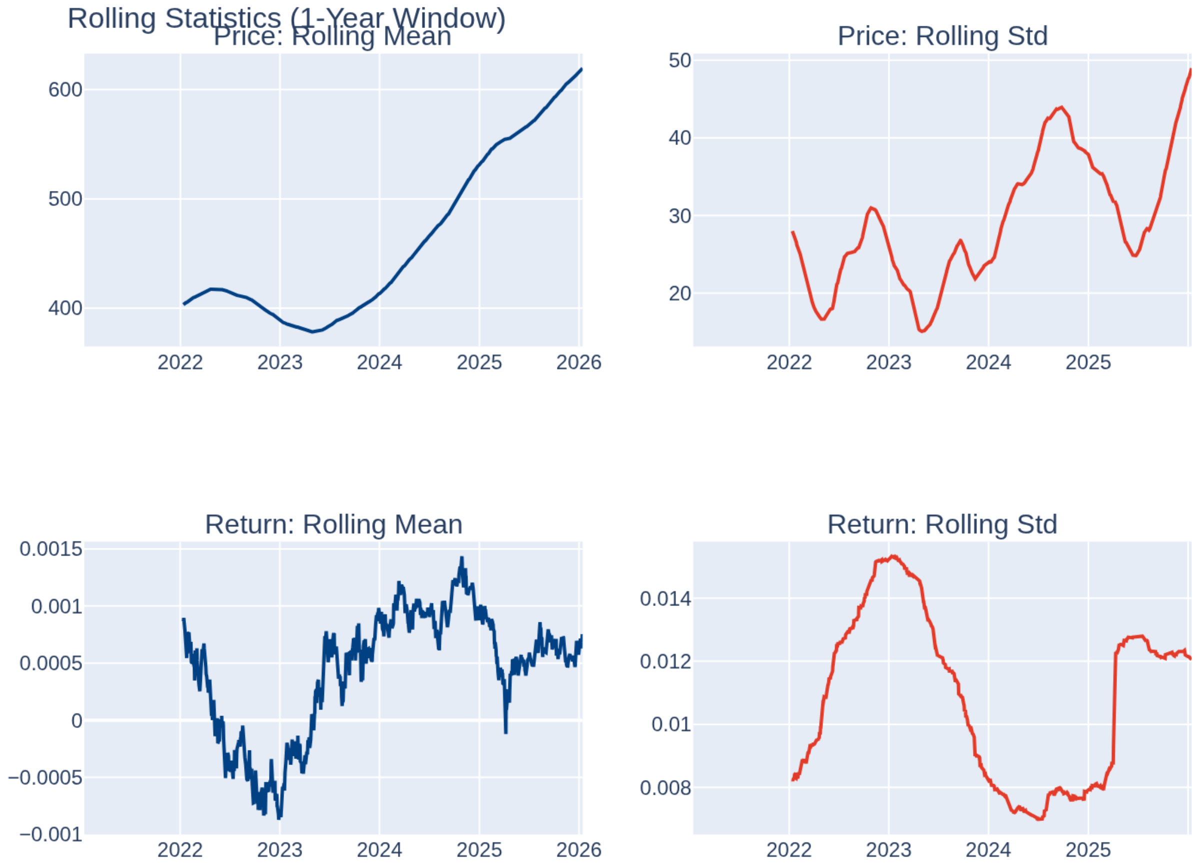
	Series	ADF Statistic	p-value	Critical 1%	Critical 5%	Stationary
0	Prices	0.4201	0.9822	-3.4356	-2.8639	No
1	Log Returns	-22.0524	0.0000	-3.4356	-2.8639	Yes

7 Visual Comparison



8 Rolling Statistics

Non-stationary series have time-varying statistics.



9 Making Series Stationary

Common transformations:

1. **Differencing**: $\Delta X_t = X_t - X_{t-1}$ (removes trend)
2. **Log transform**: Stabilizes variance
3. **Seasonal differencing**: For seasonal patterns
4. **Detrending**: Subtract fitted trend

	Series	ADF Statistic	p-value	Critical 1%	Critical 5%	Stationary
0	Prices (Level)	0.4201	0.9822	-3.4356	-2.8639	No
1	Prices (1st Diff)	-19.6430	0.0000	-3.4356	-2.8639	Yes
2	Prices (2nd Diff)	-13.6109	0.0000	-3.4357	-2.8639	Yes

10 Autocorrelation Analysis

Stationary series have autocorrelation that decays to zero; non-stationary series have persistent autocorrelation.

11 Multiple Assets

	Series	ADF Statistic	p-value	Critical 1%	Critical 5%	Stationary
0	SPY Price	0.4202	0.9822	-3.4356	-2.8639	No
1	SPY Returns	-22.0524	0.0000	-3.4356	-2.8639	Yes
2	TLT Price	-1.9540	0.3071	-3.4356	-2.8638	No
3	TLT Returns	-27.5090	0.0000	-3.4356	-2.8638	Yes
4	GLD Price	4.0599	1.0000	-3.4356	-2.8639	No
5	GLD Returns	-35.9454	0.0000	-3.4356	-2.8638	Yes
6	BTC-USD Price	-0.7409	0.8359	-3.4339	-2.8631	No
7	BTC-USD Returns	-8.1369	0.0000	-3.4340	-2.8631	Yes

12 Conclusion

Stationarity testing is a critical first step in time series analysis. Financial prices are typically non-stationary (trending, with growing variance), while returns are usually stationary. The ADF test provides a formal statistical test, and transformations like differencing or log returns can convert non-stationary series to stationary ones suitable for modeling.